Binomial Probability

In a binomial probability, there are only two possible outcomes.

The simplest example is tossing a coin – the outcomes are either heads or tails.

Consider rolling a die. There are six possible outcomes. However, we may be interested in only one in particular. Let's say we only care if a 5 shows up. Therefore, we can now call it a binomial probability because either the 5 shows up or it doesn't.

Getting the outcome we want is called a success. With our die-rolling experiment, a success is rolling a 5. The probability of success is 1/6 = 0.17, and it is designated by p.

Not getting the outcome we want is called a failure. With our die-rolling experiment, a failure is not rolling a 5. The probability of failure is 5/6 = 0.83, and it is designated by q.

$$p + q = 1$$

In a binomial experiment, we conduct a number of trials (n) and we want to know the probability of a certain number of successes (r).

The formula, which is essentially the same one from the Binomial Theorem, is:

$$P(r) = C(n,r)\,p^r q^{n-r}$$

My formula differs significantly from the book:

1. It is more common in statistics to use r instead of x.
2. It is more common to use q instead of $1-p$.
3. I'm using $C(n,r)$ instead of $\binom{n}{x}$ because the notation more clearly indicates combinations.

Binomial Expansion

A binomial is simply an expression with two terms.

Binomial expansion means we are raising the binomial to some power.

The general form is $(a+b)^n$.

$(a+b)^0$	1
$(a+b)^1$	a^1+b^1
$(a+b)^2$	$a^2+2a^1b^1+b^2$
$(a+b)^3$	$a^3+3a^2b^1+3a^1b^2+b^3$

Notice the patterns when a binomial is raised to a power:

- The number of terms is always one more than the power.
- The exponent on the a factor starts at the power and decreases by 1 to 0.
- The exponent on the b factor starts at 0 and increases by 1 to the power.
- The sum of the exponents for each term always equals the power.

Pascal's Triangle

The coefficients of each term in the expansion also follow a pattern:

$(a+b)^0$	1				1			
$(a+b)^1$	$a+b$			1		1		
$(a+b)^2$	$a^2+2ab+b^2$		1		2		1	
$(a+b)^3$	$a^3+3a^2b+3ab^2+b^3$	1		3		3		1

This pattern is called Pascal's Triangle.

Notice that each line begins and ends with a 1. The remaining entries are obtained by adding the two entries diagonally above it.