Union Rule for Probability

This is not much different than the union rule for sets:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Let's define events E and F as follows for a standard deck of cards:

E: Drawing a spade P(E) = 13/52
F: Drawing a 5 P(F) = 4/52

We want to know the probability of drawing a spade or a 5 (unions are or's, intersections are and's).

If we draw a spade, it could be a 5.
If we draw a 5, it could be a spade.

The 5 of spades occurs twice, so we have to subtract the second occurrence:

$$P(E \cup F) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Union Rule for Mutually Exclusive Events

This is not much different than the union rule for mutually exclusive sets:

$$P(E \cup F) = P(E) + P(F)$$

Let's define events E and F as follows for a standard deck of cards:

E: Drawing a spade P(E) = 13/52
F: Drawing a heart P(F) = 4/52

We want to know the probability of drawing a spade or a heart (unions are or's, intersections are and's).

If we draw a spade, it cannot be a heart.
If we draw a heart, it cannot be a spade.

$$P(E \cup F) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2}$$

Complement Rule

The probability of an event plus the probability of its complement is always equal to 1.

Recall that a complement is what is not.

$$P(E) + P(E') = 1$$

Odds

Odds are always written as a ratio of how an event can happen to how an event can't happen (its complement).

In probability, the odds in favor of an event are $P(E) : P(E')$. Odds against would be written as $P(E') : P(E)$

Odds are not always written as probabilities, but rather as a ratio of the number of ways an event can happen to how an event can't happen. Odds in favor are $n(E) : n(E')$. Odds against are $n(E') : n(E)$.

When odds in favor are written as $n(E) : n(E')$, we can calculate the probability of the event:

$$P(E) = \frac{n(E)}{n(E) + n(E')}$$

This works because the denominator is equal to the size of the sample space:

$$n(S) = n(E) + n(E')$$

Properties of Probabilities

1. The probability of each outcome must be a number between 0 and 1.
2. The sum of the probabilities of all outcomes must equal 1.