A system of equations is a set of equations that are related. Often, there are the same number of variables as there are equations – two equations, two unknowns; three equations, three unknowns; etc.

Solving the system means finding the values of the unknowns that will satisfy all of the equations. When solving a system, there are three possible outcomes:

1. We may find a single value for each variable that satisfies all of the equations. We say the system has a unique solution, and that solution is a point through which all equations pass.

2. We may find that two or more of the equations are parallel lines. We say the system is inconsistent, and that there is no solution. One way to know this has happened is if you get a statement that is false, such as $0 = 5$.

3. We may find that two or more of the equations are the same line. We say that the system is dependent, and that there are an infinite number of solutions. One way to know this has happened is if you get a statement that is true, such as $0 = 0$.

Solving a system of three or more equations gets a bit complex. A simple method is to use a matrix. A matrix consists of a row for each equation in the system:

1. The first column is the coefficients of the first variable.
2. The second column is the coefficients of the second variable.
3. The nth column is the coefficients of the nth variable.
4. The last column is the constants.

Solving a matrix is a matter of **working the diagonal**.

The diagonal of a matrix are the entries starting at Row 1 Col 1 and continuing in a downward and rightward direction. In the matrix below, the shaded entries are on the diagonal.

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 2 & -3 & 2 & 4 \\ 4 & 1 & -4 & 1 \end{array}\right]$$

The object is to work down the diagonal, starting with the first entry. For each entry, you will perform a number of row operations. The row that the entry is in is the **active** row for operations.

> For example: if you are working on the second diagonal entry of the matrix above, then your active row is the second row. The active entry is the entry of the active row that is on the diagonal.

The row operations are:

1. Change the active entry to a 1 by multiplying the row by the reciprocal of the entry. $\frac{1}{c}R_a \to R_a$
2. Change an entry to 0 by adding –1 times the entry times the active row to the row you are changing. $-cR_a + R_b \to R_b$
3. Swap two rows. $R_i \leftrightarrow R_j$

For the active row, it helps to put a square around the active entry (to indicate it needs to be a one), and a circle around the entries that need to be changed to zero.

Gaussian Elimination (to echelon form)

For each entry on the diagonal:

1. Put a square around the active entry, and a circle around all entries in the same column that are below the active entry.

2. If the entry is a 0, you may swap this row with any row **below** it. Resume with the new row as your base row.

3. Change the active entry to a 1.

4. Change each circled entry to a 0. Note: the base row does not actually change during this operation.

When you reach the bottom of the diagonal, you are in echelon form. From echelon form, you can create a triangular system and solve using back substitution.