Basic Factoring

1. Exact common factors

2. Common factors raised to different powers (factor out lowest power).

3. Fractions, even if they don't occur in all terms

Here is an example that uses all three:

$\frac{1}{2}x^5(x-3)^2(x+5)^3 - 3x^5(x-3)^3(x+5)^2$	Original problem with two terms. Each term has four factors.
$x^5\left[\frac{1}{2}(x-3)^2(x+5)^3 - 3(x-3)^3(x+5)^2\right]$	1. Exact common factors. Both terms have x^5 as a common factor.
$x^5(x-3)^2(x+5)^2\left[\frac{1}{2}(x+5) - 3(x-3)\right]$	2. Common factors to different powers. Both terms have an $(x-3)$ and an $(x+5)$. Factor out the $(x-3)^2$ and the $(x+5)^2$.
$x^5(x-3)^2(x+5)^2\left[\frac{1}{2}(x+5) - \frac{6}{2}(x-3)\right]$	3. Fractions. The first term has a fraction, so I'm going to re-write the coefficient of the second term so that it has the same denominator.
$\frac{x^5(x-3)^2(x+5)^2}{2}\left[(x+5) - 6(x-3)\right]$	3. Fractions (cont). Now I can factor out the ½ as a common factor.
$\frac{x^5(x-3)^2(x+5)^2}{2}\left[x+5-6x+18\right]$	Now a finish simplifying inside the brackets.
$\frac{x^5(x-3)^2(x+5)^2}{2}\left[-5x+23\right]$	
$\frac{x^5(x-3)^2(x+5)^2(-5x+23)}{2}$	

Factoring Formulas

Difference of two squares	$x^2 - y^2 = (x-y)(x+y)$
Difference of two cubes	$x^3 - y^3 = (x-y)(x^2+xy+y^2)$
Sum of two cubes	$x^3 + y^3 = (x+y)(x^2-xy+y^2)$

Quadratics (Trinomials)

A quadratic is an equation in which the highest power is a 2. The general form is:

$$Ax^2 + Bx + C = 0$$

Where A, B, and C are rational numbers.

The general form has three terms:

1. A squared term – the power on the variable is 2
2. A linear term – the power on the variable is 1
3. A constant term – the power on the variable is 0

We may not have all three terms, but as long as one of them has a power of 2, it is quadratic.

Be warned!! The variable may NOT be an x!

Solving Quadratic Equations

When solving quadratics, the primary skill needed is the ability to factor. Once the equation is completely factored, we apply the Zeros Theorem:

Zeros Theorem: If two or more factors equal zero, then any one of the factors could be zero.

To apply the Zeros Theorem, set each factor equal to zero and solve for the variable.

There are four cases for solving quadratic equations:

1. A is 0.

   When A is zero, the squared term is gone and we are left with a linear equation:

   $$Bx + C = 0$$

   We can easily solve this for x: $x = \frac{-C}{B}$

2. B is 0.

   When B is zero, the linear term is gone but we still have a quadratic equation:

   $$Ax^2 + C = 0$$

   We can easily solve this for x: $x = \pm\sqrt{\frac{-C}{A}}$

   Don't be alarmed with the negative sign under the radical – as long as A or C is negative (not both) the negative sign will cancel out.

3. C is 0.

   When C is zero, the constant term is gone but we still have a quadratic equation:

   $$Ax^2 + Bx = 0$$

   We can easily solve this for x by factoring and applying the Zeros Theorem:

   $x = 0$ and $x = \frac{-B}{A}$

4. A, B, and C are all not 0.

   This is more complicated, and is addressed in the next section.

Methods for Solving Quadratic Equations

First, it is helpful to have some rules for assigning signs to factors:

If the Last Term is:	**Then the Factors are both:**	**If the Middle Term is:**	**Then the Factors are:**
Positive	Same sign	Positive	Both positive
		Negative	Both negative
Negative	Opposite sign	Positive	Larger is positive
		Negative	Larger is negative

Simple Quadratic Equations: A=1
Product-Sum Method

This situation happens when the coefficient A is equal to 1.

1.	$x^2 + 5x - 24 = 0$		Set equation equal to 0.
2.	1 2 3 4	24 12 8 6	List all factors of A*C, ignoring the sign.
3.	-1 -2 -3 -4	+24 +12 +8 +6	Apply the signs to the factors according to the table above.
4.	$-3+8=5$		Identify factors that combine to equal B.
5.	$(x-3)(x+8)=0$		Write in factored form.
6.	$x-3=0$; $x+8=0$		Set each factor equal to 0.
7.	$x=3$; $x=-8$		Solve for x

Methods for Solving Quadratic Equations (cont)

Complex Quadratic Equations: A≠1
AC Variant of the Product-Sum Method

This situation happens when the coefficient A is equal greater than 1.

1.	$12x^2 - 17x - 5 = 0$		Set equation equal to 0.
2.	60		Multiply A times C.
3.	1 2 3 4 5 6	60 30 20 15 12 10	List all factors of A*C, ignoring the sign.
4.	+1 +2 +3 +4 +5 +6	-60 -30 -20 -15 -12 -10	Apply the signs to the factors according to the table above.
5.	$+3 - 20 = -17$		Identify factors of that combine to equal B.
6.	$12x^2 + 3x - 20x - 5 = 0$		Split the middle term using the factors.
7.	$(12x^2 + 3x) + (-20x - 5) = 0$		Group the first two and the last two terms together.
8.	$3x(4x + 1) - 5(4x + 1) = 0$		Factor out common multiples from each group.
9.	$(4x + 1)(3x - 5) = 0$		Factor out common binomial from both groups.
10.	$4x + 1 = 0$; $3x - 5 = 0$		Set each factor equal to 0.
11.	$x = -\frac{1}{4}$; $x = \frac{5}{3}$		Solve for x

Completing the Square

1.	$3x^2 - 6x - 1 = 0$	Set equation equal to 0.
2.	$(3x^2 - 6x) = 1$	Group the first two terms and move the constant.
3.	$(x^2 - 2x) = \frac{1}{3}$	Divide out the coefficient of the squared term.
4.	-1	Take one half the coefficient of x.
5.	$x^2 - 2x + 1 = \frac{1}{3} + 1$	Square the previous result and add to both sides of the equation.
6.	$(x - 1)^2 = \frac{4}{3}$	Factor the left side. Combine like terms on the right side.

7.	$x-1=\pm\frac{2}{\sqrt{3}}$	Take the square root of both sides.
8.	$x=\pm\frac{2}{\sqrt{3}}+1$	Solve for x.

Quadratic Formula

This is obtained by starting with the General Form and using "Completing the Square" to solve for x.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Identify a, b, and c.
Plug them into the formula.
Solve for x.

The Discriminant

This is the part of the quadratic formula that is underneath the radical sign.

Calculate its value to determine how many solutions there are to the equation.

If it is:	Then there are/is:
greater than 0	two distinct real solutions
equal to 0	exactly one real solution
less than 0	no real solutions