Multiple Events with Multiple Outcomes

Determining how many ways a series of events can happen.

By the Fundamental Counting Rule, just take the number of outcomes for all events and multiply them together:

(Outcomes for Event A) x (Outcomes for Event B) x …

Ordering Objects

Let's say we have a collection of objects and want to know how many ways we order them. How many ways can we arrange five different books on a shelf? We'll use a modification of the multiplication rule.

For the first book, we can choose any one of the five books.

However, for the second book we can only choose from one of four books because one has been chosen already.

So, the number of ways to order the five books is given by:

5 x 4 x 3 x 2 x 1 = 120

Notice the pattern: for each succeeding position, there is one less choice. This pattern is called factorials, and is designated $n!$, where n is the number of objects to be arranged.

Combinations

Let's say that the number of places available is less than the number of objects. For example, you have five books, but only space on the shelf for three of them. How many ways can you do that?

If it does not matter what order they appear, then the order (Math, Chemistry, English) is not different from the order (Chemistry, English, Math).

The formula for combinations is:

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

We are calculating the number combinations of n objects r at a time.

On the calculator, the function is ${}_nC_r$.

Permutations

Imagine we do care about the order of the books. It matters that the chemistry book is placed after the math book, or vice versa.

We can truncate the calculation above, stopping after we've placed the three books:

5 x 4 x 3 = 60

The formula for permutations is:

$P_{n,r} = \dfrac{n!}{(n-r)!}$, where n is the number of objects that you have, and r is the number that you can place.

We are calculating the number permutations of n objects r at a time.

On the calculator, the function is ${}_nP_r$.

Distinguishable Permutations

Suppose your objects are not distinct.

When you list all possible arrangements, some will be identical.

3 Objects: A, A, C

6 Permutations: AAC, ACA, CAA, CAA, ACA, AAC

But, we can't distinguish between the first A and the second A. This is calculated by:

$$\frac{n!}{n_1!n_2!n_3!\cdots n_k!}$$

where:

n is the total number of objects
n_1 is the number of objects of the 1st kind
n_2 is the number of objects of the 2nd kind
n_3 is the number of objects of the 3rd kind
n_k is the number of objects of the kth kind