Multiple Events with Single Outcomes

The same event is repeated two or more times. Each time, we are looking for a particular outcome, although the outcome does not have to be the same for each event.

The events will be either independent or dependent.

Independent Events

Let's say we want to roll two dice. We'll define Event A as rolling a 3 on the first die, and Event B as rolling a 5 on the second die.

The probability of Event A is 1/6, and the probability of Event B is 1/6. Rolling the first die has no effect on what happens to the second die – so the two events are independent.

Using the multiplication rule:

P(A and B) = P(A) x P(B) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

For two independent events: P(A and B) = P(A) x P(B).

This also works for more than two events: P(A and B and C) = P(A) x P(B) x P(C).

Dependent Events

Let's now consider drawing two cards from a standard deck. We'll define Event A as drawing a 3 on the first card, and Event B as drawing a 5 on the second card.

With Replacement

If we draw the first card, observe it, then put it back into the deck before drawing the second card, then the two events remain independent. The probability of drawing a 3 on the first card is 4/52. Because we put the card back, we still have 52 cards from which to draw. Therefore, the probability of drawing a 5 on the second card is 4/52.

So, the probability of drawing a 3 and 5 is given by the formula above:

$$\frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13} = \frac{1}{169} = 0.0059$$

Dependent Events (cont)

Without Replacement

If we draw the first card and set it aside, then we have one less card to select from when we draw the second card. The probability of drawing a 3 on the first card is 4/52. With one less card, the probability of drawing a 5 on the second card is changed, depending on what was drawn on the first card.

If the first card is not a 5, then the probability of getting a 5 on the second card is 4/51, because we have one less card to draw from.

If the first card is a 5, then the probability of getting a 5 on the second card is 3/51, because there is also one less 5 in the deck.

We can still calculate the probability of the two events occurring the way we want it to occur:

$$\frac{4}{52}\times\frac{4}{51}=\frac{1}{13}\times\frac{4}{51}=\frac{4}{663}=0.0060$$