Binomial Expansion

A binomial is simply an expression with two terms.

Binomial expansion means we are raising the binomial to some power.

The general form is $(a+b)^n$.

$(a+b)^0$	1
$(a+b)^1$	a^1+b^1
$(a+b)^2$	$a^2+2a^1b^1+b^2$
$(a+b)^3$	$a^3+3a^2b^1+3a^1b^2+b^3$

Notice the patterns when a binomial is raised to a power:

- The number of terms is always one more than the power.
- The exponent on the a factor starts at the power and decreases by 1 to 0.
- The exponent on the b factor starts at 0 and increases by 1 to the power.
- The sum of the exponents for each term always equals the power.

Pascal's Triangle

The coefficients of each term in the expansion also follow a pattern:

$(a+b)^0$	1	1
$(a+b)^1$	$a+b$	1 1
$(a+b)^2$	$a^2+2ab+b^2$	1 2 1
$(a+b)^3$	$a^3+3a^2b+3ab^2+b^3$	1 3 3 1

This pattern is called Pascal's Triangle.

Notice that each line begins and ends with a 1. The remaining entries are obtained by adding the two entries diagonally above it.

Factorials

A factorial is the product of the natural numbers. Recall that the natural numbers are 1, 2, 3, … etc.

Factorials are denoted with the "!". So, 5! means 5x4x3x2x1, which equals 120. Note that the 1 can be left off, as it does not affect the final answer.

Binomial Coefficient

There is a far more efficient way of calculating the coefficient for a particular term:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

where:
n is the exponent of the binomial expansion.
r is the term number you wish to calculate the coefficient for.

This is actually the formula for combinations.

n^{th} Term of a Binomial Expansion

Book version	My version
The n^{th} term of $(a+b)^n$ is $\binom{n}{n-r} a^r b^{n-r}$ where: n is the exponent a is the left term b is the right term r is the "looping variable" When fully expanding a binomial, r starts at n and decreases by 1 for each successive term.	The n^{th} term of $(a+b)^n$ is $\binom{n}{r} a^{n-r} b^r$ where: n is the exponent a is the left term b is the right term r is the "looping variable" When fully expanding a binomial, r starts at 0 and increases by 1 for each successive term. Note: r is always 1 less than the term number.

Binomial Expansion Shortcut

Example: $(x-3)^3$

1.	Draw a horizontal line for each term of the expansion (always one more than the exponent).	$\_ + \_ + \_ + \_$
2.	In the middle of each line, write the "a" term in parenthesis. Then, place an exponent on each, starting with the exponent of the expansion for the first term and decreasing by on for each successive term.	$\underline{(x)^3} + \underline{(x)^2} + \underline{(x)^1} + \underline{(x)^0}$
3.	To the right of the "a" term, write the "b" term in parenthesis. Then, place an exponent on each, starting with the zero for the first term and increasing by on for each successive term.	$\underline{(x)^3(-3)^0} + \underline{(x)^2(-3)^1} + \underline{(x)^1(-3)^2} + \underline{(x)^0(-3)^3}$
4.	To the left of the 1st term, put a 1.	$\underline{1(x)^3(-3)^0} + \underline{(x)^2(-3)^1} + \underline{(x)^1(-3)^2} + \underline{(x)^0(-3)^3}$
5.	Use the 1st term to get the coefficient for the 2nd. Multiply the coefficient by the exponent of the "a" factor and divide by 1: $1\times 3 \div 1 = 3$	$\underline{1(x)^3(-3)^0} + \underline{3(x)^2(-3)^1} + \underline{(x)^1(-3)^2} + \underline{(x)^0(-3)^3}$
6.	Use the 2nd term to get the coefficient for the 3rd. Multiply the coefficient by the exponent of the "a" factor and divide by 2: $3\times 2 \div 2 = 3$	$\underline{1(x)^3(-3)^0} + \underline{3(x)^2(-3)^1} + \underline{3(x)^1(-3)^2} + \underline{(x)^0(-3)^3}$
7.	Use the 3rd term to get the coefficient for the 4th. Multiply the coefficient by the exponent of the "a" factor and divide by 3: $3\times 1 \div 3 = 1$	$\underline{1(x)^3(-3)^0} + \underline{3(x)^2(-3)^1} + \underline{3(x)^1(-3)^2} + \underline{1(x)^0(-3)^3}$
8.	Simplify.	$x^3 - 9x^2 + 27x - 27$

The process for calculating the coefficient of the **next** term is:

1. Start with the coefficient of the current term.
2. Multiply that by the exponent of the "a" factor of the current term.
3. Divide that by the term number of the current term.