

Chapter 7	Matrices and Determinants
Section 3	Inverses of Matrices and Matrix Equations

Recall that in the Gauss-Jordan elimination, the primary matrix is reduced to a matrix with only 1's and 0's. The 1's are on the primary diagonal which starts at entry (1,1). All of the remaining entries are 0. A matrix in this form is called an **identity** matrix.

Multiplying any matrix by an identity matrix results in the original matrix.

The **inverse** of a matrix is any matrix such that multiplying it by the original results in an identity matrix. In other words, a matrix multiplied by its inverse results in an identity matrix.

To find the inverse of a matrix:

1. Construct a doublewide matrix, separated down the middle by a line. The left side of the line is the original matrix; the right side of the line is an identity matrix with the same dimensions.
2. Use Gauss-Jordan elimination to reduce the left side to an identity matrix. Be sure to apply all operations to both sides of the doublewide matrix.
3. The right side will now be the inverse of the original matrix.

Solving Matrix Equations

Recall that when solving for x , you always do the inverse of what is being done to x :

- If a constant is being **added** to x , **subtract** that constant from both sides.
- If x is being **multiplied** by a number, **divide** both sides by that number.
- If x is under a **square root** sign, **square** both sides.
- If x is in the **exponent**, take the **log** of both sides.

The pairs of terms in each line above represent inverse operations of each other.

To solve a matrix equation, multiply both sides of the equation by the inverse of the coefficient matrix.

A matrix equation of size N contains three matrices:

1. An $N \times N$ matrix of the coefficients.
2. An $N \times 1$ matrix of the variables.
3. An $N \times 1$ matrix of the constants

The left side of the equation is the product of the coefficient matrix and the variable matrix. The right side of the equation is the constant matrix.