A matrix consists of a row for each equation in the system:

1. The first column is the coefficients of the first variable.
2. The second column is the coefficients of the second variable.
3. The n^th column is the coefficients of the n^th variable.
4. The last column is the constants.

Solving a matrix is a matter of **working the diagonal**.

The diagonal of a matrix are the elements starting at Row 1 Col 1 and continuing in a downward and rightward direction. In the matrix below, the shaded elements are on the diagonal.

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 2 & -3 & 2 & 4 \\ 4 & 1 & -4 & 1 \end{array}\right]$$

The object is to work down the diagonal, starting with the first element. For each element, you will perform a number of row operations. The row that the element is in is the **active** row for operations.

> For example: if you are working on the second diagonal element of the matrix above, then your active row is the second row. The active element is the element of the active row that is on the diagonal.

The row operations are:

1. Change the active element to a 1 by multiply the row by the reciprocal of the element. $\frac{1}{c}R_a \rightarrow R_a$
2. Change an element to 0 by adding –1 times the element times the active row to the row you are changing. $-cR_a + R_b \rightarrow R_b$
3. Swap two rows. $R_i \leftrightarrow R_j$

For the active row, it helps to put a square around the active element (to indicate it needs to be a one), and a circle around the elements that need to be changed to zero.

Gaussian Elimination (to echelon form)

For each element on the diagonal:

1. Put a square around the active element, and a circle around all elements in the same column that are below the active element.

2. If the element is a 0, you may swap this row with any row **below** it. Resume with the new row as your base row.

3. Change the active element to a 1.

4. Change each circled element to a 0. Note: the base row does not actually change during this operation.

When you reach the bottom of the diagonal, you are in echelon form. From echelon form, you can create a triangular system and solve using back substitution.

Gauss-Jordan Elimination (to reduced echelon form)

There are two methods to complete this process.

Method 1

Complete the Gaussian Elimination to echelon form.

For each element in the diagonal, starting at the **bottom** and working **up**:

Place a square around the element, and a circle around all numbers in the same column above the element. Change each circled element to a 0. Note: the base row does not actually change during this operation.

When you reach the top of the diagonal, you are in reduced echelon form.

Method 2

For each element on the diagonal:

1. Put a square around the active element, and a circle around all elements in the same column that are above and below the active element.

2. If the element is a 0, you may swap this row with any row **below** it. Resume with the new row as your base row.

3. Change the active element to a 1.

4. Change each circled element to a 0. Note: the base row does not actually change during this operation.

When you reach the bottom of the diagonal, you are in reduced echelon form. The solution can be read directly from the last column.