

A rational function is a fractional function with polynomials in the numerator and denominator.

With fractional functions, we have asymptotes.

An asymptote is a line that the function approaches infinitely close to, but (usually) never touches.

Asymptotes come in three varieties:

1.	Vertical Never crossed.	Vertical asymptotes are vertical lines that pass through an x-intercept. The x value is constant along the line; therefore, the slope is undefined. Vertical asymptotes happen whenever the denominator equals zero. Set the denominator equal to zero and solve for x. Draw vertical lines through those x values.
2.	Horizontal May be crossed once.	Horizontal asymptotes are horizontal lines that pass through a y-intercept. The y value is constant along the line; therefore, the slope is 0. Horizontal asymptotes depend on the degree of the numerator and denominator. Let n be the degree of the numerator, and d be the degree of the denominator: <ul style="list-style-type: none"> • If $n < m$, then the horizontal asymptote is $y=0$. • If $n=m$, then the horizontal asymptote is $y=a_n/b_m$, where a_n is the lead coefficient of the numerator, and b_m is the lead coefficient of the denominator. • If $n > m$, there is no horizontal asymptote <p>Note that the graph may cross a horizontal asymptote.</p>
3.	Slant Never crossed. May not be a line.	Slant asymptotes are slanted lines that have both x and y intercepts. Slant asymptotes happen when the degree of the numerator is greater than the degree of the denominator. The equation for the slant asymptote is obtained by dividing the denominator into the numerator. The quotient (ignoring the remainder) is the equation for the slant asymptote.

End Behavior

The sign and oddness/evenness of the leading term determine the end behavior for slant asymptotes. Otherwise, it is a matter of testing values.

Guidelines for Sketching

1.	Find the intercepts – write as points.				
	x:	Set the numerator equal to zero and solve for x.			
	y:	Make a fraction of the constants only from the numerator and denominator.			
2.	Find the asymptotes – write as equations.				
	v:	(vertical) Set the denominator equal to zero and solve for x.			
	h:	(horizontal)			
		If degree of numerator	<	degree of denominator	$y=0$
		If degree of numerator	=	degree of denominator	$y=\text{fraction of lead coefficients}$
s:	(slant)				
	If degree of numerator	>	degree of denominator	$y=Q(x)$	
3.	Test points.				
	Select two points on both sides of each vertical asymptote.				
4.	Plot points, sketch asymptotes, sketch graph.				