Chapter 4	Polynomial and Rational Functions
Section 3	Real Zeros of Polynomials

Mind Your p's and q's

A zero of a polynomial is simply a number which, when plugged in for the variable, causes the polynomial to evaluate to 0.

To find the real zeros of a polynomial, one technique is to list all possible zeros, then divide each one into the polynomial to see if it works. If it divides evenly (a remainder of 0), then the number is a zero of the polynomial.

1. List all of your p values. These are factors of the constant.
2. List all of your q values. These are factors of the leading coefficient.
3. List all possible combinations of ±(p/q).
4. Start dividing.

Once a zero is found, the original equation can be re-written using the divisor as a factor and using the quotient line as the reduced polynomial.

It is often easier to continue the search for zeros using the reduced polynomial. Once it is down to quadratic level, use one of the techniques for solving quadratics.

An Example

	$P(x) = x^3 + 10x^2 + 31x + 30$	Original problem
1.	1, 2, 3, 5, 6, 10, 15, 30	List all p values – factors of 30
2.	1	List all q values – factors of 1
3.	+1, +2, +3, +5, +6, +10, +15, +30 –1, –2, –3, –5, –6, –10, –15, –30	List all possible combinations of ±(p/q)
4.	$\begin{array}{r\|rrrr} 1 & 1 & 10 & 31 & 30 \\ & & 1 & 11 & 42 \\ \hline & 1 & 11 & 42 & 72 \end{array}$	First attempt: +1 The last number in the quotient line is not zero, therefore +1 is not a zero of the polynomial.
5.	$\begin{array}{r\|rrrr} 2 & 1 & 10 & 31 & 30 \\ & & -2 & -16 & -30 \\ \hline & 1 & 8 & 15 & 0 \end{array}$	Second attempt: –2 The last number in the quotient line is a zero, therefore +2 is a zero of the polynomial.
6.	$P(x) = (x-2)(1x^2 + 8x + 15)$	Re-write the polynomial using the divisor as a factor and the quotient line as the reduced polynomial.

Descartes Rule of Signs

This rule helps to determine the number of possible real zeros.

- The number of variations in sign of P(x) is the maximum number of positive zeros (zeros with a value greater than zero).

- The number of variations in sign of P(-x) is the maximum number of negative zeros (zeros with a value less than zero).

The important concepts to understand here are:

1. Variation in signs:

   A variation in signs occurs when two adjacent terms have opposite signs.

2. Less than by an even whole number (because of the conjugate zeros theorem):

   Once you count the number of variations, keep subtracting 2. Each time you subtract 2, you get a count for the possible number of zeros. Example: if you have 5 variations in sign for P(x), then you could have 5, 3, or 1 positive zeros.

3. P(-x):

   Replace x with –x in the polynomial and simplify. This is the exact same process as testing for symmetry.

The Upper and Lower Bounds Theorem

This theorem helps to eliminate the number of possible zeros to be tested.

1. Divide the polynomial by a possible zero.
2. If the remainder is 0, then it is a zero.
3. Look at the signs of all terms in the quotient line:
   a. If all of them are positive, then the zero is the largest zero; you do not need to test any possible zeros greater than this one.
   b. If they alternate positive and negative, then the zero is the smallest zero; you do not need to test any possible zeros less than this one.