

Chapter 4	Polynomials and Rational Functions
Section 2	Dividing Polynomials

Long Division

In any division, you have four parts:

- Dividend – the number being divided
- Divisor – the number doing the dividing
- Quotient – the number of times the divisor goes into the dividend
- Remainder – the number left over if the divisor does not go evenly

Example: 8 (dividend) is divided by 5 (divisor). 5 (divisor) goes 1 (quotient) time with a remainder of 3 (remainder).

$$\begin{array}{r} \text{quotient} \quad \text{remainder} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

The same principle applies to polynomials, but the process is a bit more complex.

It is VERY important that all missing powers of x are filled in with $0x^n$, where n is the missing power.

1. Divide the first term of the divisor into the first term of the dividend. Write the term as the first term of the quotient.
2. Multiply the first term of the quotient by every term in the divisor, writing each resulting term below the appropriate term of the dividend.
3. Subtract the result from Step 2 from the dividend (or, change the signs and add).
4. Divide the first term of the divisor into the second term of the dividend. Write the term as the second term of the quotient.
5. Multiply the second term of the quotient by every term in the divisor, writing each resulting term below the appropriate term of the result from Step 3.
6. Subtract the result from Step 5 from the result from Step 6 (or, change the signs and add).
7. Repeat the pattern until the subtraction results in a power less than the divisor.

Synthetic Division

Synthetic division uses the coefficients from the dividend and the constant from the divisor.

The setup looks something like this:

$$\begin{array}{r|l}
 \text{constant} & \text{coefficients} \\
 & \hline
 & \text{intermediate calculations} \\
 & \hline
 & \text{quotient and remainder}
 \end{array}$$

For synthetic division, the divisor is always a linear factor.

The process is:

1. Enter the constant
2. Enter the coefficients, using a 0 for any missing power of x .
3. Bring the first coefficient to the quotient line.
4. Multiply the constant by the coefficient and write the result on the intermediate line under the second coefficient.
5. Add the numbers in the second column and write the result in the quotient line.
6. Repeat the pattern for each coefficient.

Remainder Theorem

You can evaluate what the remainder will be by plugging the constant into the original polynomial and evaluating.

Lets say you have a polynomial $P(x)$ being divided by $x-c$. Evaluate $P(c)$. The result would be the remainder as if you had done the division.

Factor Theorem

Doesn't help us much. Basically, if $x-c$ divides evenly into $P(x)$ (ie: the remainder is zero), then $x-c$ is a factor and c is a zero.