Simple Linear Inequalities

In a simple linear inequality, the = is replaced by any one of the following inequality signs: <, ≤, >, ≥, or ≠. There are still two sides to the inequality, just as there are two sides to an equation.

When solving a simple linear inequality, the goal is to isolate the variable on the left side. The rules for solving equations apply, but there are two additional rules to bear in mind:

1. When multiplying or dividing by a negative number, change the direction of the inequality sign.
2. **<u>Never</u>** multiply or divide by a variable.

Complex Linear Inequalities

In a complex linear inequality, there are **three** sides to the inequality. There are two inequality signs in any combination.

When solving complex linear inequalities, the goal is to isolate the variable on the middle side. All rules for solving simple linear inequalities apply, with one extension:

Whatever you do to one side, you have to do to all three sides.

Sketching on a Number Line and Interval Notation

1. Draw a number line.
2. Mark each solution on the number line.
3. If the sign for a solution is:
   a. < or >, draw an open circle on the solution
   b. ≤ or ≥, draw a closed circle on the solution
4. From each open or closed circle, draw an arrow pointing:
   a. Left for < or ≤
   b. Right for > or ≥
5. Below the solution:
   a. If open circle and arrow points left, write)
   b. If closed circle and arrow points left, write]
   c. If open circle and arrow points right, write (
   d. If closed circle and arrow points right, write [
6. On the inside of the parenthesis or bracket, write the solution.
7. On the other side of the solution, write a comma.
8. On the other side of the comma, if there is not another solution:
   a. (-∞ if the left side of the comma is blank.
   b. +∞) if the right side of the comma is blank.

	Number Line	Interval Notation
<	○ ←	)
>	○ →	(
≤	● ←	]
≥	● →	[

Solving nonlinear inequalities is more complicated:

1. Move all terms to the left side.
2. Factor the left side as completely as possible. Note: if all terms are divisible by a constant, divide out, rather than factor out, that constant.
3. Set each factor equal to 0, and solve.

Using the results from Step 3, we are now going to draw a sign chart.

1. Draw a number line and plot those values. The values divide the number line into intervals.
2. Draw a vertical line through each point plotted. You will now have a column for each interval.
3. On the first line of each column, just below the number line, write in a test value to use for that interval.
4. Draw a vertical line on the left side of the left-most column.
5. On the left side of the new line, list all factors of the inequality, one per row, starting just below the line of test values.
6. You now have a table, similar to a Microsoft© Excel© spreadsheet or a Microsoft© Access© table. Each cell is identified by the interval-column and factor-row.
7. For each interval, plug the test value into each factor, and write the sign of the result in the cell.
8. At the bottom of each interval-column, write the sign of the result:
    a. If there are an even number of negative signs, then the result is positive.
    b. If there are an odd number of negative signs, then the result is negative.
9. For each interval, compare the resulting sign against the inequality sign:
    a. If the inequality sign is $<$ or $\leq$, then intervals with a negative result are part of the solution.
    b. If the inequality sign is $>$ or $\geq$, then intervals with a positive result are part of the solution.
10. For all intervals in the solution set, write in interval notation using the correct $\cup$ (union) and/or $\cap$ (intersection) signs as needed.

Solving Inequalities Graphically

The first step is to treat it like a regular equation, and graph that equation.

The second step is to pick a test point and plug it into the inequality.

- If you get a true statement, then shade the area on that side of the graph.
- If you get a false statement, then shade the area on the other side of the graph.

For a test point, if the equation does not pass through the origin, then use that as your test point because it's the easiest.