

Chapter 2	Weighted Voting Systems: The Power Game
Section 2	The Banzhaf Power Index

Terminology

Coalition: Any set of players that might join forces and vote together.

Weight of the coalition: The total number of votes controlled by a coalition.

Winning coalition: A coalition with enough votes to win a motion.

Losing coalition: A coalition without enough votes to win a motion.

Grand coalition: The coalition consisting of all players.

Critical player: A player whose desertion from a coalition changes it from a winning to a losing coalition.

Banzhaf Power Index: The percent of winning coalitions for which a particular player is a critical player.

Banzhaf Power Distribution: A complete listing of the Banzhaf Power Indexes.

Notation

Each player is assigned a set of initials representing either their position or their name. A coalition is listed as a comma separated list of these initials enclosed in curly brackets {}.

Finding the Banzhaf Power Index of a Player

1. Make a list of all possible coalitions
2. Classify the coalitions as winning or losing.
3. In each winning coalition, underline the players who are critical players.
4. Count the number of times a particular player is critical.
5. Count the total numbers of times all players are critical.

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Calculating the Number of Coalitions

This is from basic probability – computing the number of subsets in a set.

For each element in a set, we have two choices: either include it in a subset or don't include it. We have to make a choice for each element in the set – each choice is a stage.

In probability, we calculate the total number of ways something can happen by taking the number of choices at each stage, and multiplying them together.

Our number of choices for each stage equals two. The number of stages equals the number of elements in our set.

Let N equal the number of elements in the set. We are multiplying two times itself N times. This is 2^N , which represents the number of subsets.

However, this also includes what is known as the empty subset – the subset with no members. Obviously, this is not a valid coalition, so it has to be subtracted off. Our formula for the number of coalitions becomes:

$$2^N - 1$$